LIBERÁLNÍ INSTITUT

THE DECLINE AND ASCENT OF THE EXPECTED UTILITY THEORY

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1. Economics and the Life-World

If the situation of the expected utility theory is so dramatic that it is necessary to express it in categories of "rise" and "fall" (as W. Leinfellner does)[1], then the only method of the philosophical scrutinizing of this theory consists in the search for its fundamental ontological grounds.

The term "fundamental ontology" suggests that it is the phenomenological method which seems to be the most acceptable approach to the problem. In order to justify this methodological approach, we must expound the philosophical background of the conception of utility as developed in neo-classical economics and especially in its Austrian version.

In accordance with a generally accepted opinion, the concept of utility is derived from utilitarianism of Jeremy Bentham; it is widely known that in his version of the "felicific calculus" we can find the anticipation of the basic principles of the conception of marginal utility. However, it is not so widely known that the original version of the Austrian economics as can be found especially in Carl Menger (and also in Ludwig von Mises) is based upon Aristotelian philosophy.

Some of the Aristotelian principles immanently "functioning" in Menger's economics can be expressed as follows:

1) The world exists, independently of our thinking and reasoning activities.

2) There are in the world certain simple 'essences' or 'natures' or 'elements' as well as laws, structures or

connections governing these, all of which are strictly universal.

3) Our experience of this world involves in every case both an individual and a general aspect.[2]

As Menger puts it: "...the goal of research in the field of of theoretical economics can only be the determination of the general essence and the general connection of economic phenomena".[3]

Menger's methodology of economics oriented towards the recognition of the general essence of economic phenomena is at variance with the purely mathematical treatment of economics which is derived from the ideas of René Descartes, the great critic of Aristotle. "Austrian economics, in contrast, is marked by a willingness to sacrifice both the goal of predictive power and the mathematical tools associated therewith" precisely in order to come to an understanding of "the basic everyday categories in which the science of economics has its roots".[4] In harmony with this Aristotelian spirit of the Austrians, the mathematical apparatus of neo-classical economics (e.g. the curves of indifference etc.) can be admitted only as arising from primarily non-mathematical categories of pre-scientific, commonsensical, everyday economic life.

It should be stressed that the Aristotelian methodology of Austrian economics was considerably influenced by the works of Franz Brentano (1838 - 1917) who is regarded as one of the founding fathers of the Austrian philosophical tradition.[5] Brentano's Aristotelianism made also a clear impact on the phenomenological philosophy of his famous pupil Edmund Husserl

(1859 - 1938).

Husserl's phenomenological method is closely connected with his doctrine of the life-world (Lebenswelt), which is in some aspects similar to the Aristotelian methodology of Menger. According to this doctrine, the idealized objects of modern (= mathematical) scientific knowledge are derived from the everyday, pre-scientific life-world sphere, i. e. from the sphere of the immediately evident personal experiences. It means that natural laws as expressed in mathematical form by means of various equations, formulae, curves etc. are products of idealizing activities of our thinking. Naturally, Husserl does not deny the existence of regularities in nature; he only emphasizes that these regularities are primarily found in the immediate experience of the everyday pre-scientific world in which - and this is especially important - we are experiencing ourselves as free beings able to choose between different alternatives. Husserl states that the usual interpretation of science, according to which the idealized (mathematical) objects of scientific knowledge are a purely objective and impersonal basis of our personal experiences in our everyday life, implies the impossibility of human freedom, which leads to naturalization of man and, in turn, to the crisis of European culture (manifesting itself in political manipulation and in various forms of totalitarianism).

According to Husserl, the task of phenomenological method with regard to the idealized objects of such sciences which use mathematical tools consists in finding the original,

pre-scientific, commonsensical or everyday sense of these idealized objects through their "re-animation" in immediate experience in the frame of the life-world;[6] this re-animation should unveil the derivative character of the mathematically expressed objects of scientific knowledge in relation to the life-world which is their forgotten basis. In harmony with Menger and Husserl we can state that utility is a constitutive part of our pre-scientific life-world. The point of question of all theories of utility including the expected utility theory is the problem of the measurement and quantification of utility; we can say that it is the problem of the transformation of the phenomenon of utility as immediately given in our pre-scientific personal experience into an idealized, quantifiable and objectively valid object of scientific knowledge.

2. The Conception of von Neumann and Morgenstern

According to von Neumann-Morgenstern, the basic situation, in which the quantity of utility manifests itself empirically, in an objectively valid empirical form, is not the interaction amongst the participants of free market (as it is in the theories of marginal utility)[7], but the situation of our decision-making. An expressed decision as an empirically testable verbal behaviour[8] is an exteriorized manifestation of the inner act of the preferring of the utility of some action to another. The preferability of the utility of any action can be known only

ex post as the result of an accomplished act of preference. This result is exteriorized in an verbal act or in a real action realizing the preferred alternative. The quantity of utility does not manifest itself in the pure actuality of the process of the actual satisfaction of needs, but in the anticipation of the desirable future, which is the specifically human type of acting.

According to von Neumann-Morgenstern, the means of the quantification of utility is probability. The introduction of the dimension of probability into the situation of decision-making can be held for the unveiling of such aspect of this situation in which the quantitative dimension of preferring can appear. In case of non-probabilized decision-making this quantitative dimension of preference exists, too; but, it appears only as relative quantity expressed in the words "higher than", "lower than" and "equal". It implies that in case of decision-making between certain alternatives, the absolute quantity of utility is only an unimportant characteristic.

After the introduction of probability, preference ceases to be a function with three possible values (>, <, =); it becomes a function characterized by the continuum of functional values, which is the result of the application of probability with its continuum of functional values. Thus, probability becomes the numerical foothold of the expected utility theory.[9]

Probability is connected with utility only in the consciousness of decision-maker, namely in the form of the combining (=multiplying) of two possible utilities u, v with their corresponding probabilities α, $1 - \alpha$; this operation is

defined by von Neumann-Morgenstern's axiom (3:1:b):

$$w(\alpha.u + (1 - \alpha).v) = w(u) + (1 - \alpha).w(v).^{10}$$

Naturally, the operation of the combining of utilities with probabilities has no external, empirically testable correlate in objective reality as such;[11] the empirical manifestation of this correlation is verbal behaviour (= the verbal expression of the decision) and/or practical action.

In von Neumann-Morgenstern, the basis of the operational quantification of utility is the subjective experience of the relation of indifference between the quantity of the utility of certain alternatives and the quantity of the utility of merely probable alternatives. The subjective experience of indifference as transformed into mathematical relation "=" is the starting point of the calculation of the numerical values of the utility function.[12] E.g., on the basis of a system of equations like

$$A = (1 - \alpha).B + \alpha.C$$

(A is the utility of a certain expectation, B and C are the utilities of probable alternatives, α is the probability of the forthcoming of an uncertain event characterized by the utility C) we can calculate the quantities od A, B, C.

3. The Life-World Basis of Probability

But what enables us to connect probability with utilities? What are the conditions of the possibility of the measuring of utility by means of probability? What is the pre-scientific or commonsensical basis of probability? In von Neumann-Morgenstern, the idea of the multiplying of the numerical values of utility with probabilities is a product of pure intuition; it is presented without deeper philosophical grounds, in a form precisely characterized by Hegel's word "aus der Pistole erschossen".

The conditions of the possibility of the quantification of utility by means of probability consist in the latent temporal character of probability. (Naturally, what is meant is objective probability defined by the frequency of the occurrences of some event.) Being expressed numerically, the temporal character of probability seemingly disappears; nevertheless, it exists in a hidden, forgotten or 'sedimented' form if we put it in the language of phenomenology.

If we say that some event has an objective probability (e.g. $p = 1/5$), the meaning of our statement is the same as the meaning of the statement according to which the considered event actually occurs just once in the time which corresponds to five possible repetitions of its occurrence. The definition of objective probability presupposes the existence of some regularities in the repeated occurrences of the considered event. These regularities appear only with respect to the Law of Large

Numbers; the quantitative expression of these regularities can be stated only on the basis of this law.

The possibility of the comparing of the probabilities of different utilities presupposes a hidden ideal condition according to which the repeated occurrences of events with these utilities are related to an identical unit of time: The certainty of the occurrence of some event ($p = 1$) means that this event will occur just once in a considered time interval. The probability $p = 1/2$ means that an event with this probability will occur just once in the time interval which is the double of the time interval in which the event with probability $p = 1$ will occur just once. Only in this way (i.e. through the mediation of certain /=guaranteed/ occurrences in a considered time unit and through the mediation of the corresponding multiples of this unit) it is possible to relate probabilities to its empirical basis.

What is the pre-scientific character of the time the immediate experience of which is preserved or 'sedimented' in our understanding of probability? It is the time which separates the living subject of a need from its object, i.e. it is the negatively, unpleasantly experienced time of a passive waiting for the occurrence of some event. (The time of waiting differs from the negatively experienced active disutility of labour, which is also expressed in the form of the quantity of time).

Hence it follows that that waiting cannot be related to the occurrence of the object of primary, i. e. biological needs. We can wait only if our primary needs are satisfied. Thus, any

passive waiting for the unification of the subject and object of a need cannot be related to the preservation of our biological existence. That is why the time of waiting cannot be identified with the qualitatively determined time of the experience of the gradating pressure of biological needs; it is only the negatively experienced abstract homogeneous time reduced to its quantity.

The proper domain of such waiting or expecting are games and the free-market enterprise characterized by the regularities which manifest themselves in the form of the contingent occurrences of events. It is the attribute of modern civilization that not only the economic life but all the world of human social and spiritual activities is subordinated to the play of contingency expressed by statistical regularities. The lot of the contemporary Western man, ingeniously described by Samuel Beckett in his *Godot*, is waiting. (The only exception is the automation of the satisfying of biological needs.)

In von Neumann-Morgenstern's axiom (3:1:b), the negatively experienced quantity of the time of waiting expressed numerically in the values of probability is connected with the utility of the considered event. In the situation of decision-making and -realizing, the disutility, unpleasantness or negativity of the time of waiting is especially apparent, because the expectation of the utility for which we have decided excludes the realization of other utilities. E.g. in case of an event with utility U and probability $p = 1/n$, the corresponding quantity of the negatively experienced time of waiting is equal to n hypothetical time units. In the process of decision-making, the quantity of the

negatively experienced time of waiting manifests itself through reducing the numerical utility n times (= U/n). This is the right sense of the multiplication of probability by numerical utility. Or, to put it in a more understandable way, for the sake of the acquiring of a large utility we are willing to sacrifice a very large quantity of the negatively experienced time of waiting; this quantity corresponds to a low probability. For the sake of the acquiring of a small quantity of utility we are willing to sacrifice only a small quantity of the negatively experienced time - we choose this utility only if its probability is very high.

In von Neumann-Morgenstern, utility is measured by the relative quantity of the negatively experienced time of waiting which we are willing to sacrifice for the sake of the acquiring of the considered utility. The comparative character of quantity of the negatively experienced time of waiting results from the fact that the numerical value of probability is constituted by the ratio of the multiples of the hypothetical unit of negatively experienced time. If we wanted to measure utility by any absolute time unit, then this unit would represent different degrees of disutility for the same man in different situations.

The identification of probability with the ratio of the quantities of the negatively experienced time of waiting is equal to the rejection of the possibility of the existence of the absolute unit of utility (so called "util"). Consequently, the numerical expression of utility by means of numerical values of probability is in fact only a quasi-numerical expression.[13]

Von Neumann-Morgenstern's axiom (3:1:b) expresses the mathematical connection of the abstract, homogeneous, negatively experienced time of waiting with corresponding utility. In case of the possible occurrence of some event with expected utility U and probability $p = 1/n$ $[n \in \langle 1,\infty)]$, the decision-making subject behaves in the same manner as in case of the guaranteed occurrence of the reduced utility which arises by multiplying the numerical utility U with the probability p (the reduced utility $U_r = U/n$). Thus, the possible discrete occurrences and non-occurrences of an event with utility U in long run are transformed into a quasi-real occurrence of an event with reduced utility. The possible non-occurrence of the event manifests itself as a proportional reduction of its utility.

This transformation can be held for the point in which von Neumann-Morgenstern's theory deviates from the real course of the process of decision-making so radically that it cannot be used as a normative model of human decision-making.

The real process of decision-making under risk and uncertainty is not referred to the guaranteed occurrence of an event with utility U in a small or large interval of homogeneous time; it proceeds with regard to the possible non-occurrence of that event and with regard to all implications resulting from it. In von Neumann-Morgenstern, the possible occurrence (resp. non-occurrence) of some event is not regulated by the principle "all or nothing". They postulate a ficitional continual transition from all to nothing. Nothingness is presented in the form of the proportional reduction of the utility of the "all".

The quantitative reduction of an utility through its being combined with its probability masks the substantial difference between our relation to certain (guaranteed) alternatives and the subjective experience of the decision-making process under risk and uncertainty. The specific character of this experience results from the possibility of the non-occurrence of an event with a desirable utility, i.e. from the invasion of nothingness. Thus, the comparatively expressed quantity of abstract, homogeneous, negatively experienced time corresponding to some event with utility U must be submitted to some additional subjective transformation in our consciousness. Otherwise, probability as the 'sedimented' form of the experience of that time could not be applied to decision-making under risk and uncertainty which is oriented to the impending future comprehending the possibility of the non-occurrence of desirable utility. Von Neumann-Morgenstern's theory does not reflect this very important fact. Consequently, the main direction of the famous Allais' criticism of the American School is oriented toward this weak point of von Neumann-Morgenstern's theory.

4. Allais' Criticism of the American School

The starting point of Allais' criticism is an empirically verified statement according to which the human behaviour under risk and uncertainty does not correspond to the axioms of von Neumann-Morgenstern. Allais states that our human psychic nature prefers the security of the acquiring of a small utility in the

neighbourhood of certainty to a large utility in the neighbourhood of large risk.[14] Consequently, prudent (or anxious) man prefers a small utility with a high probability of occurrence to a large utility with a low probability of occurrence, though the numerical product arising from the multiplication of the quantified small utility with the high probability is equal to the numerical product of the large utility and the low probability (this later product can be even smaller than the former one). Similarly, for the man who loves risk, the subjective attractiveness of a large utility with a low probability is larger than the subjective attractiveness of the small utility with a high probability, though the numerical values of the corresponding products are equal; these products can be even in the relation which is inverse to the real relation of the risk lover's preference. Hence it follows that von Neumann-Morgenstern's type of combining the probability with utility in the form of their numerical product cannot play the assigned role in the real process of decision-making. The real structure of the transformation of a given probability (it is not important if this probability is objective or subjective - see Chapter 7) and a given utility into an accomplished decision and into practical behaviour is much more complicated than that simplified regularity expressed by von Neumann-Morgenstern's axiom (3:1:b).

According to Allais, the structure of the considered transformation is determined by different types of the dispersion of the probabilities of the occurrences of events with different

utilities in neighbourhood of the statistical mean value of probabilities. What is meant by Allais is the case of single possible events comprehended in some of possible complex alternatives of acting.[15] Prudent (or anxious) man prefers the small statistical dispersions of probabilities, the risk lover prefers the large ones. In mathematical expression, the subjective attractiveness {= Allais' subjective value s(V)} is described by the function h which is a function of the density of the probability of the occurrence of utility:

$$s(V) = h[\ \ (\ )]$$

/h[( )] is the density of probability/.

According to Allais, the subjective attractiveness (or the subjective psychological value) s(V) based on the density of probability is the fundamental element which plays the "absolutely substantial" role in decision-making and acting under risk and uncertainty.[16] In Allais' theory, the type of dependence of decision-making on the type of the density of probability is an individual characteristic of human subject (*matière des gouts*).[17] We agree with these Allais' conclusions, but there appears a consequential problem. Expressing the psychic transformation of probability and utility, Allais does not emphasize sufficiently that this transformation arises not only in case of the complex alternatives which comprehend many mutually excluding events, but also in case of simple alternatives comprehending only the possible occurrence of only

one event. Our (and also Allais') analyses of the behaviour of the prudent man and the risk lover concern just the simple alternatives which are not able to be described by the term "density of probability". Consequently, the structure of the psychic or subjective transformation of probability and utility in the process of decision-making which concerns complex alternatives must necessarily be derived from the structure of psychic transformation corresponding to simple alternatives; namely, the latter structure is primary and determining from the viewpoint of the evolution of our cognitive and evaluative activities. This is why our further analyses will concern the structure of the psychic or subjective transformation of probability and utility in the process of decision-making which concerns simple alternatives.

5. Determinedness to Act

Our examples of the different types of decision-making show that the subjective attractiveness s(V) arises from the understanding of the uncertainty of the occurrence of some event in the context of the established individual system of values (it is the aspect of the *esse* of individual human being) and with respect to momentaneous situation (it is the aspect of the *existere*). Besides utility and probability, the constitutive elements of subjective attractiveness of an action are such psychic features as the pleasure of gambling, toleration toward risk and the opposite of this toleration - the degree of

anxiousness etc.

In accordance with the basic methodological principles of empirical sciences, which acquire the operationalization of all subjectively experienced data, the only available interpretation of Allais' function s(V) expressing the subjectively experienced attractiveness of some probabilized utility must be based upon the relation of subjective processes to their objective manifestations.

An objectivized result of the inner acts of preferring which concern alternatives with different degrees of subjective attractiveness can be seen in our verbal behaviour in which we use the words "better than", "worse than", "indifferent" etc. But, the original, pre-scientific purpose of the considered verbal behaviour is the expression of the degree of one's inner determinedness to possible practical acts oriented towards the realization of the corresponding alternatives. Namely, the sense of decision-making consists in the realization of such alternative which amounts to the maximum of subjective attractiveness s(V). Generally speaking, subjective attractiveness is fully objectivized only in practical act.

From the strictly empirical or objective standpoint, the understanding of subjective attractiveness s(V) must arise from its objective manifestations; it implies that subjective attractiveness s(V) must be re-defined as the degree of (one's) determinedness to act. Determinedness to act (D_a) is that individual psychic state which is the last link of the chain of the various subjective processes which cause practical act. It

can be also said that determinedness to act is the synthesis of various determinations which influence our practical acts. The above mentioned strictly objective viewpoint implies that free decision-making is identical with the choosing (and realization) of such alternative which evokes the highest degree of determinedness to act.

Thus, Allais' function s(V) expresses numerically the degree of individual determinedness to act which is the realization of some risky alternative of possible acting. More precisely, it is the degree of individual determinedness to act, evoked by an alternative which comprehends a possible desirable event with utility U and probability p. Generally, the function s(V) expresses numerically the degree of determinedness to act concerning the alternatives which comprehend mutually excluding events with probabilities $p_1 \ldots p_n$, and utilities $U_1 \ldots U_n$. The degree of determinedness to act D_a can be also characterized as the degree of toleration with respect to risk or as the inversely expressed degree of axiousness.

The exact mathematical expression of the functional relation of the degree of determinedness to act D_a to given values of probability and utility and to one's individual psychic characteristics are very complicated for the very reason that Allais regards both utilities and probabilities as subjective data. The problem consists in the fact that the only exact form of the quantification of subjective experiences can be derived from the Weber-Fechner Law which expresses the relation of subjective reaction to objective stimulus.

For the sake of the application of a modification of the Weber-Fechner Law we must suppose that the degree of determinedness to act is characterized by a higher degree of subjectivization than we can find in subjective probability and subjective utility. However, this presupposition is in accordance with Allais' regarding the function s(V) as the subjective transformation of probability and utility. With respect to the higher degree of subjectivization in case of D_a, subjective probability and subjective utility appear as relatively objective data. Naturally, in further phases of our inquiry it will be necessary to objectivize also p and U as far as possible.

The exact character of the Weber-Fechner Law is based on the specific methodological application of differential thresholds which manifest themselves in the process of perception of the degrees of the intensity of subjective qualities. Applying differential thresholds to the quantification of subjective qualities, the repetition of identical objective units, which is the basis of measurement and quantification, is replaced with the repetition of the identical subjective experience of the appearing of a discernable quantitative difference in perceived quality, i. e. with the repetition of the act of the becoming aware of perceived difference *in statu nascendi;* in the sphere of flowing inner experiences, only this act can be identical in its repetitions.

Let us investigate the differential thresholds in the subjective experience of the quantitative changes of the degree (= intensity) of determinedness to act in case of simple

alternatives.

If we consider that a possible event with utility U and probability p evokes a degree of determinedness to act D_a, the following statement can be held for evident:

If the given degree of determinedness to act D_a is experienced as high and if U is constant, it is necessary to experience only a very small increment Δp of probability in order that a discernable increment ΔD_a (=differential threshold) of the subjectively experienced intensity of determinedness to act may arise. Similarly, if the degree of determinedness to act is experienced as low (i.e. if anxiety is experienced very intensively) and if U is constant, it is necessary to experience a considerably large increment Δp of probability in order that the discernable increment of the subjectively experienced intensity of D_a may arise. Thus, if the utility U is constant, the experienced quantity (intensity) of determinedness to act D_a and the increment Δp of probability are inversely proportionate with regard to the possibility of the experience of the discernable increment of the intensity of determinedness to act D_a. These evident relations can be mathematically expressed by the following equation:

$$\Delta D_a = k \cdot D_a \cdot \Delta p \qquad (1)$$
$$(k = \text{const.})$$

We know that $D_a = f(p,U)$.[18] Nevertheless, as we could see, the individual psychic reaction to those data is caused only by

probability.

Namely, utility is experienced in the acts of subjective evaluation in such a way that it is not possible to imagine any higher degree of its subjectivization. In the situation of non-probabilized decision-making the subjectively experienced utility is not quantitatively different from the corresponding degree of determinedness to act; in case of probabilized decision-making, the probability of an event does not influence its utility in itself (the "*esse*" of our hierarchy of values does not depend on the uncertainty of the possible "*existere*" of events and things). Consequently, the subjectively experienced D_a must be proportional to subjectively evaluated utility.

In contrast to utility, probability is able to become an object of a further psychic adaptation. Even if probability in Allais is a subjective datum, its subjective character consists in the subjective character of the cognitive acts of estimating. Having been subjectively estimated in the cognitive acts, probability can and must be subjectively experienced also in the "further" evaluative acts. Only probability has a scope for a further psychic adaptation.

This character of probability results mathematically in:

$$D_a = D_a(p) \cdot U \qquad (2)$$

After the substitution into (1) we obtain:

$$D_a(p + \quad p) \cdot U - D_a(p) \cdot U = k \cdot D_a(p) \cdot \quad p \cdot U \qquad (3)$$

and hence:

$$D_a = k \cdot D_a \cdot \quad p \tag{3'}$$

If we substitute differentials for increments, we obtain:

$$\frac{dD_a}{dp} = k \cdot D_a \tag{4}$$

The solution of this equation is:

$$D_a(p) = C \cdot e^{k.p} \tag{5}$$

($C \neq$ const.)

The equation (5) is an exact mathematical expression of the fact that in the process of the subjective experience of probability every experienced numerical value of probability is transformed into the corresponding individual degree of determinedness to act.

More precisely, D_a is only that portion of the total determinedness to act, which is caused by the subjectively evaluated probability of some alternative; the other portion of the total determinedness to act D_a is caused by the utility of the considered alternative.

In order to ascertain the numerical values of the constants k and C we cannot use - as it has been said above - any direct "physical" method resulting in the ratio type of scale. (In case

of a "physical" treatment, the constants k, C would be equalizing constants with regard to the different systems of the measurement units used in the measurement of probability and D_a.)

The numerical values of the constants k and C (which express the individual character of one's subjective adaptation of known values of probability) can be ascertained empirically on the ground of the results of the explored person's preferential behaviour under risk and uncertainty. It can be said that the individual (or personal) character of one's subjective adaptation of known values of probability manifests itself very explicitly in the (practical) activity of betting. If somebody places a bet, he in fact loses the pertinent amount of money; in order to ascertain k and C, we can identify his loss with the actual obtaining of the negative utility resp. disutility $(- U_1)$ which is (only for him!) numerically equal just to the amount of money he has betted. The term "actual" means that the considered obtaining of the disutility $(- U_1)$ is certain, i.e. that the probability of its occurrence is equal to 1.

Naturally, everybody who places a bet knows that there is a probability p of the gaining of the (positive) utility U_2 the numerical value of which is larger than the absolute numerical value of the disutility $(- U_1)$. (We can suppose a gamble characterized by previously known and definite values of probability, e.g. dice.)

Thus, somebody is willing to stake 20 USD $(- U_1)$ so as to be allowed to hope in the gain 200 USD (U_2) the probability of which is known to him as p (e.g. $p = 1/50$). Now, let us investigate the

degree of his willingness to staking. E.g., we can find that, in unchanged circumstances (U_2 = 200 USD, p = 1/50), he is willing to stake only 3 USD more (- U_1 = 23 USD) so as to be allowed to participate in the gamble; but, he is not willing to stake 24 USD. We can even suppose that our man, having unpremediatedly staked 24 USD, would try to cancel his bet if he became fully acquainted with the conditions of the gamble. This trying can be regarded as resulting from negative determinedness to (certain) act. It implies that, for our investigated person, the bet larger than 23 USD and lower than 24 USD together with the other determinations (U_2,p) results in the zero degree of determinedness to (certain) act. Thus, we can say that our person, having staked 23 USD, reaches **the marginal degree of determinedness to act** in the considered circumstances.

In order to ascertain the numerical values of the constants k and C, we must deal with the zero degree of determinedness to act, which is characterized by the fact that the sum of the numerical expressions of all determinations which affect the investigated person is equal to zero. It concerns the certainty of the actual disutility (- U_1) as well as the probability of the possible utility U_2. The fact that the "marginal" bet is placed in the interval (23, 24) USD will be expressed by means of a virtual increase δU_1 as related to the disutility (- U_1).

Thus, we are obtaining the following equation:

$$U_2.C.e^{kp} - (U_1 + \delta U).C.e^{k.1} = 0, \qquad (6)$$

and hence

$$k = \ln \frac{U_2}{U_1 + \delta U_1} \cdot 1/(1 - p) \qquad (7)$$

We can see that the magnitude of the constant C (which has disappeared) can be arbitrary; it implies that the magnitude of C is indifferent with regard to the preferential behaviour of individual person. For all our further investigations, we can appoint C as equal to 1 (C = 1).

Consequently, it is the magnitude of the constant k which expresses the individual character of one's subjective adaptation of the known values of probability; the magnitude of k constitutes the principally individual, personal dimension of determinedness to act under the conditions of risk and uncertainty; or, in other words, it is the individual reaction to the mere probability (i.e. uncertainty) of a desirable event which finds its expression in k. (Returning to the model situation mentioned above, we can say that, in similar circumstances, some other person would be willing to stake maybe 27 USD - it would be caused by the corresponding difference in magnitudes of k in both persons.)

The correspondence between the numerical value of probability and the "probabilistic portion" of the degree of determinedness to act D_a is unequivocal (i. e. it can be characterized as a one-to-one correspondence). Nevertheless, the numerical expression of the D_a is not measurable directly (= through an empirical operation using any objective units of

measurement); the different magnitudes of the constant k express only differences in human preferential behaviour which is contituted by the relations "better than", "worse than" or "equal". It implies that the problem can be formulated as follows: What scale type can be applied to the measuring of D_a? It implies that we are looking for such group of transformations which is the highest one in the hierarchical order of the groups of transformations preserving the unequivocal character of the numerical valuation of D_a.

It might appear that the succession of the numerical values of D_a amounts merely to an ordinal scale; it would imply that this succession is unequivocal (or invariant) up to the izotonic group of transformations $x' = f(x)$. But - on the other side - we know that the values of D_a are determined by an unequivocal functional relation to probability. It suggests that it is possible to submit the succession of the values of D_a to some more complicated transformation. Of course, this transformation must preserve the substantial characteristic of the function $D_a(p)$, i.e. it must preserve the fact that the linear increase of the value of probability corresponds to the exponential increase of the value of D_a. Or, in other words, the first derivative of the function $D_a(p)$ must be invariant to this transformation.

It is evident that the transformation

$$x^* = x + \beta, \qquad \beta = 0 \qquad (8)$$

$$(\beta = \text{const.})$$

conforms to all above mentioned conditions. This transformation

refers to so called "difference scale".[19]

It is very useful for us to submit the formula (5) for D_a to such transformation: namely, as a result of this operation we can have $D_a = 0$ if $p = 0$.

For this purpose we can determine

$$\beta = -1 \tag{8'}$$

so that

$$D_a^* = D_a - 1 = e^{k.p} - 1 \tag{8''}$$

After substituting the transformed shape D_a^* for D_a in the formula (2) we have

$$\boxed{D_a = (e^{k.p} - 1) \,.\, U} \tag{9}$$

Complex alternatives can be described by the extending of the formula (9):

$$D_a = [\exp(k.p_1) - 1].U_1 + [\exp(k.p_2)].U_2 + \cdots + [\exp(k.p_n)].U_n \tag{10}$$

(n = the number of events comprehended in a complex alternative).

We think that the formula (9) expresses the psychological law the structure of which is similar to the structure of the Weber-Fechner Law. But, the functional relation between the

subjective and objective aspect of decision-making as expressed in (9) is inverse with regard to the Weber-Fechner Law: instead of logarithmic dependence we have the exponential one.[20]

The formula (9) describes man's relation to anticipated utility under risk and uncertainty; accordingly, it does not refer to the course of actual stimulation as the Weber-Fechner Law does. D_a is in fact inversely proportionate to the degree of anxiety which is a necessary constitutive part of our interestedness in our own being; this anxiety which is experienced especially under risk and uncertainty has a strong tendency to diminish itself, if there is any sufficient reason for it. This sufficient reason consists usually in the obtaining of such information about future events, which in certain degree validates our hopes and expectations. After reaching this individually different degree of validation, the anxiety starts to diminish very rapidly (according to the exponential curve), which is equal to the rapid increase of determinedness to act. The reason for this rapidity is a delight arising from the decline of anxious tension. This delight reaches its highest intensity at the end of the process of the gradation of the validation of subjective hopes. This is in contrast to the process of the satisfaction of needs, where the highest intensity of delight is at the beginning of consumption.

On the other hand, if the obtained information does not yield the individually needed degree of validation of the hopes and expectations and if the degree of validation diminishes more and more, the above mentioned anxiety increases very rapidly. By

means of this rapid increase, the anxiety fills its function which consists in the averting of risky actions. The increase of anxiety enables thus the further existence and self-realization of man.

It is the possibility of non-occurrence of some desirable event (= the possibility of the invasion of nothingness, expressed by probability) that evokes anxiety; it should be stressed that the increase of anxiety is much more larger than the corresponding decline of the probability of the desirable event. Thus, the formula (9) expresses quantitatively the inseparability of man's orientation to the future from his anxious interestedness in his own being.

It should be mentioned that, starting from the known values of probability and from the known magnitude of the constant k, we are able to co-ordinate a cardinal number to every subjectively experienced utility (the technical way of the reckoning of this numbers is similar to the method used in von Neumann-Morgenstern). But, von Neumann and Morgenstern assumed that the cardinal numbers co-ordinated to utilities through the mediation of their method are objectively valid because of the objective (intersubjective) validity of the calculus of probability. We believe that our investigations have shown that such type of the objective quantification of utility is not possible. With regard to the individual character of the constant k we can say that the cardinal numbers co-ordinated to various utilities through the mediation of probability and its function D_a^* express only individual person's reaction to risk and uncertainty and,

therefore, they cannot be objectively valid.

6. Attitudes to Risk

If we compare the course of the exponential curve determined by (8) with the course of the straight line defined by the equation $D_a(p) = p$, we can acquire three basic types of human attitude toward risk. (The competent section of the line $D_a(p) = p$ corresponding to $p \in \langle 0,1 \rangle$ is the graphic expression of the dependence of D_a on probability in von Neumann-Morgenstern, where $D_a(p) = p$.) Let us remind that D_a signifies the numerical values of that portion of determinedness to act, which is caused by the subjective experience or adaptation of probability.

The first type of attitude toward risk (Fig. 1) is characterized by the fact that the determinedness to act D_a is for every considered probability larger than the numerical value of this probability. This characteristic can be expressed still otherwise: the expected utility of each of all possible values of a considered stimulus is always larger than the expected utility of the statistical mean value of this stimulus, which can be described by the formula $U(S) > U(ES)$.[21] Accordingly, this characteristic concerns the type of pure risk lover.

Contrariwise, Fig. 3 represents a very high measure of anxiousness. Each of the numerical values of determinedness to act is lower than the corresponding numerical value of probability. This is a description of a very high degree of aversion to risk. Similarly, we can characterize this type by the

formula $U(S) < U(ES)$ which covers all possible values of stimulus S. It should be added that the shape of the exponential curve is determined by the magnitude of the constant k which expresses individual attitude toward risk.

The third type of human attitude toward risk (Fig. 2) is proper to the absolute majority of people. This type is characterized as follows: The exponential curve intersects the straight line in the point P_T (p_T, D_{aT}). The value of p_T is determined by the equation:

$$\exp (k.p_T) - 1 = p_T \qquad (11)$$

(Naturally, p_T depends on the magnitude of the constant k.)

If $p > p_T$, then $D_a > p$ (D_a increases more rapidly than p); if $p < p_T$, then $D_a < p$ (D_a falls more rapidly than p).

Accordingly, the formula (9) enables us to express the whole continuum of possible attitudes to risk; anxious man and pure risk lover are the extreme poles of this continuum. Using the formula (11), we can determine the very important point P_T which separates the sphere of toleration to risk from the sphere of aversion to risk.

7. Allais' Concept of Subjective Probability

The second main object of Allais' criticism of the American School is its conception of probability as operating with the

notion "relative frequency". In accordance with this conception, the values of probability are derived from the *ex post* measurements of the past preferential behaviour of individuals.[22] These measurements concern frequencies, which implies that probability is frequency in a long run. Allais' criticism seems to be fully legitimate, because von Neumann-Morgenstern's model in fact "turned out to be a normative, prescriptive and Platonistic model"[23] based on the normative subordination of the present and future behaviour to the past preferences as expressed numerically by the values of objective probability.

In this context we agree with Leinfellner who stresses that "the Neumann-Morgenstern axioms worked in all riskless game-theoretical model, where the rules of the game enforce a normative frame on the game, but turned out to be a castle in the air in decisions under risk and uncertainty."[24] For the sake of the precise expression of the real course of decision-making under risk and certainty, Allais introduces a subjectivist concept of probability. We can reconstruct Allais' theory in the following way: Probability is a subjective notion which is (intentionally) related to the supposed frequency of the occurrences of some event; this notion does not arise from the *ex post* measurements of the real past occurrences of that event, but it arises in the subjective cognitive acts of estimating, comparing etc. Allais admits the participation of various irrational psychic motives (*confiance en étoiles*) in the forming of subjective probability, but the result of all these acts is

nothing but a supposed frequency. He separates the level of the deformation of probability in subjective cognitive acts (so called element II - this deformation is equal to the subjective estimation of probability) from the level of the deformation of probability in the subjective acts of evaluation (element IV.).

The subjective character of probability enables us to relate the notion of probability even to isolated events which are (or will be) realized for the first time; in case of such isolated events, objective probability cannot be applied.[25] The recognition of subjective probability should enable us to describe theoretically not only our behaviour directed by deductive reason, but also our creative intelligence, the main factor of decision-making under risk and uncertainty.[26]

Naturally, in harmony with Allais, we must admit that the cognitive acts of estimating as mentioned above in connexion with our analyses of determinedness to act are usually influenced by irrational factors, especially in decision-making which concerns complex alternatives of economic and political activity. However, it should be stressed that there is a basic difference between the acts of subjective (irrational) estimation of probability and the subjective evaluative acts which are oriented toward this probability as having been previously estimated. In any case, Allais' conception of subjective probability must be seriously taken into consideration.

But, unfortunately, the most revolutionary part of Allais' theory - the subjectivist concept of probability which should ruin the von Neumann-Morgensternian castle in the air - is also

the weakest point of his system: Subjective probability as conceived in Allais is not measurable in principle (let us recall the *confiance en étoils*).

8. The Measurement of Subjective Probability

The only way of the preservation of Allais' theoretical contribution as a whole consists in finding a way of operationalization of the notion of subjective probability. This operationalization is possible only on the basis of the relation of subjective probability to objective probability.

The initial model of this relation can be shown in the experimental situation in which objective probability of the occurrence of some event exists and is measurable (e.g. it is a gamble), but the investigated person does not know its value; of course, the investigator knows it. The task of the investigated person consists in the estimation of the unknown value of objective probability. (E.g. by comparing the considered event which is new and unknown for the explored person with analogous events the probability of which is known etc.) In order to discharge this task, the investigated person utilizes its psychic faculties: intelligence, theoretical knowledge, experience.

On the ground of such experimental situation we can try to outline a skeleton of the relation of subjective probability to the objective probability. The mathematical expression of this relation is:

$$|p_O - p_{Est}| = \frac{h \cdot N}{I^m \cdot K^n \cdot E^q} \qquad (12)$$

(p_O - objective probability, p_{Est} - subjectively estimated probability, I - the IQ of investigated person, K - the degree of the investigated person's knowledge of social reality, E - the quantitatively expressed extension of his experience; the constants m, n, q quantify the participation of single psychic faculties in decision-making; N - the degree of the novelty of a considered event (the measurability of novelty which is a complex of the new, up to this time non-experienced aspects of an event seems to be very problematic, but, in using some indirect and approximative methods, it is not excluded in principle), h - the constant which expresses the measure of the irrationality of the investigated person's behaviour - e.g. a high value of h corresponds to excessive self-confidence based on the *confiance en étoiles.*)

The formula for subjectively estimated probability is:

$$p_{Est} = p_O \pm \frac{h \cdot N}{I^m \cdot K^n \cdot E^q} \qquad (13)$$

If we express the utility U by the Weber-Fechner Law:

$$U = \log \frac{C_S + M}{C_S} \qquad (14)$$

(C_s = starting capital, M = the monetary value of the amount in which we are interested).

After the substitution of (14) and (13) into (10), we obtain the synthetic formula:

$$D_a = \{\exp [k.(p_{O1} \pm \frac{h.N_1}{I^m.K^n.E^q})] - 1\} \cdot \log \frac{C_s + M_1}{C_S} + ...$$

$$... + \{\exp [k.(p_{On} \pm \frac{h.N_n}{I^m.K^n.E^q})] - 1\} \cdot \log \frac{C_s + M_n}{C_S} \quad (15)$$

Supposing that psychologists dispose (or will dispose) of suitable indirect methods of operationalization of the factors I, K, E (and also of the factor N), we can state that the formula (15) would enable us to correlate the fully developed, non-reduced complexity of the subjective dimension of decision-making to the objectively measurable data which amount to the objective conditions of decision-making.

9. Some Methodological Implications

But what is the methodological meaning of that mathematical description of the relation between subjective and objective dimension of human decision-making?

The basic field of the application of (15) are the experimental situations in which the investigator knows the

values of p_0, N, I, K, U and takes interest in the values of the constants k, h, m, n, q which are quantitatively different for every individual; the object of psychologist's interest are individual psychic dispositions which determine the behaviour of certain individual under risk and uncertainty. But the basic field can be extended; although the value of the constant k is a personal characterisitc of individual, it can be supposed that there exists a regular statistical distribution of these values in human population. We must also apprehend the fact that the statistical distribution of the values of k etc. changes with respect to different social groups and classes (this is very outstanding in case of k which expresses the differences in subjective attitudes to risk and uncertainty) and also with respect to the historical conditions (= the changing economic, moral, religious and other influences).

The finding of the distribution of individual psychic attitudes to risk and uncertainty in the whole of population is possible only in the frame of an extensive psycho-sociological investigation based on the representative assortment of investigated persons.

Apart from the ascertainment of the correlations of the constants k, h etc. to social groups and classes, the result of the investigation would consist in the determination of the numerical interval in which the quantity of the subjective deviation from objective probability oscilates; the extension of this interval is probably also conditional on the individual's membership in his social group.

Nevertheless, is there any possibility of the application of this project in economic or political practice? Is it possible to pass from an experiment, in which utility and probability are known, to real life?

Concerning utility, the way of its measuring through the mediation the of the Weber-Fechner Law presupposes investigator's being acquaintanced with the property situation of investigated person; the method of von Neumann-Morgenstern is not conditional on this aquaintance, but it presupposes the cognition of the results of the past acts of preferring, which are the starting point of the *ex post* measurement of utility. In addition to it, the axiom (3:1:b) does not correspond to reality because it prescribes the same attitude to risk for all people. The formula (15) enables an *a posteriori* measurement of utilities, too. (Naturally, if we know all other constants and variables). In case of (15), the exactness of measurement is much higher than in case of that oversimplified (3:1:b).

With regard to the fact that attitude towards risk depends on individual's membership of group and class, it seems to be more suitable to apply the Weber-Fechner Law to the measurement of utility, because it characterizes the utilities as referred to property situation.

The problem of knowledge of objective probability seems to be more difficult. Instead of the *a priori* known probabilities as usually applied to experimental situations, in the investigation of real economic life we must presuppose the knowledge of objective probability which can be obtained through the mediation

of measurement and statistical methods (in case of the unchanging rules of game) or through the mediation of the theoretical predictions of the future evolution of the objective probabilities of the occurrence of various events (this is valid in case of a new type of events in changing circumstances). Naturally, a theoretical prediction can be merely an approximation to the real course of events, but it is much more exact than a subjective estimation.

If we could know the interval in which the subjective deviation from objective probability usually oscilates and if we could know the theoretically determined objective probability of some event, we could also find the statistical dispersion of the values of subjectively estimated probability. Consequently, the application of such formulae as (15) would enable us to forecast the character of the corresponding preferential behaviour.

Naturally, the knowledge of the style of people's reactions to uncertainty can concern just their reactions to that "authentic" uncertainty which is the immanent dimension of the spontaneous order of the free market system and liberal democracy. It implies that it is necessary to discern between this knowledge which can be developed only in the frame of open society and the fatal conceit of constructivist rationality. The ultimate sense of the investigation of the different types of statistical distribution of the values of k, h etc. in their historical dynamism would be the exact predictions of the economic and political behaviour of social groups, classes and of the whole of society. Accordingly, the aim of this

investigation cannot be seen in the substitution of constructivist rationality for spontaneous order; it can be seen only in the studying of the conditions of the functioning of this order for the sake of making them more perfect.

10. The Concluding Remark

The positive side of the theory of von Neumann-Morgenstern consists in the measurability of expected utility; but, this positive side is redeemed by the loss of complexity in the conceiving of human activity. Contrariwise, Allais emphasizes the complexity of the psychic dimensions of human decision-making under risk and uncertainty; but, he resigns the measurability of probability. Consequently, the further ascent of the expected utility theory can be based only on the synthesis of all positive elements of both theories.

Notes

1) See Leinfellner (1991), pp. 84-98.

2) See Smith (1990), pp. 4-8.

3) Menger (1883), p. 7., resp. Menger (1981), p. 37.

4) Smith (1990), p. 25.

5) Cf. Smith (1986), pp. 1-36.

6) Cf. Husserl (1954), Beilage III.

7) Cf. Böhm-Bawerk (1886), II.3.

8) Cf. Savage (1954), Chapter Nine.

9) Neumann-Morgenstern (1947), p. 19.

10) Ibid., p. 24.

11) Cf. Berka (1974), p. 138.

12) Ibid., p. 133.

13) Ibid., p. 170.

14) Allais (1953), p. 538.

15) Ibid., p. 510.

16) Ibid., p. 511.

17) Ibid., p. 521.

18) Cf. Allais (1953), p. 521.

19) Cf. Pfanzangl-Baumann-Huber (1968), pp. 28ff, 74ff, 97ff; Berka (1972), p. 43; Berka (1977), p. 175,

20) D_a is a psychic state which is characterized by a higher degree of subjective complexity than the subjective consciousness of p and U. The "distance" of the consciousness of p and U from objectivity is smaller than the "distance"

of D_a. It can be said that p and U are relatively "more objective" than D_a.

21) Bureš (1988), p. 22.

22) Cf. Leinfellner (1991), p. 88.

23) Ibid.

24) Ibid.

25) Cf. Allais (1953), p. 508.

26) Leinfellner (1991), pp. 95-97.

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